YAROSLAV TAGAMLITZKI

SELECTED PAPERS

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PREFACE

This volume of selected papers of Yaroslav Tagamlitzki (1917–1983), one of the most influential and colorful Bulgarian mathematicians, is published to celebrate the centenary of his birth. Tagamlitzki never acted as a boss and did not have political power; his great influence on the development of the mathematical sciences in Bulgaria came through his mathematical ideas and by his unprecedented dedication to teaching and upbringing young mathematicians.

The order of the papers follows approximately the time of publication. The most important papers, originally published in Bulgarian or Russian, are translated into English in order to make them accessible to the international mathematical community. The papers, published in German and French are reproduced in their original language.

The volume begins with the PhD thesis of Yaroslav Tagamlitzki, prepared in Leipzig University (1943) under the supervision of Paul Koebe (1882–1945), a leading expert in complex analysis, known for his fundamental results on the uniformization of Riemann surfaces. After this substantial work in complex analysis, the young Tagamlitzki turns to a different direction, working mostly on functional and real analysis. He published in a short time (1946–1952) a series of papers in Doklady AN SSSR, C. R. Acad. Sc. Paris and in Bulgaria, addressing different problems, but united by the idea of building a new method for studying such problems. One of the most important papers in this series is devoted to the problem of Abel-Goncharov. Yaroslav Tagamlitzki was awarded the National Prize of science for this work. Let us mention the fact that some of the papers in this series are presented for publication by such legendary figures in the world of mathematics as Andrei Kolmogorov, Sergei Bernstein and Paul Montel.

In the next period, Tagamlitzki's interests were focused on the notion of extreme points, called *indecomposable* elements, in general functional spaces. He then proved his famous *Theorem on the geometry of cones* which provides a unified approach for deriving a considerable part of the results he has achieved after his PhD thesis and many of the results of his students. Several years the *Theorem on the geometry of cones* was in the center of interest of the young mathematicians around Yaroslav Tagamlitzki and he was happy to see various applications of his theorem by his students. A real personal disappointment came when Tagamlitzki found out that his *Theorem* is a consequence of Krein-Milman's theorem, proved in 1940. As a result Tagamlitzki lost interest in searching for other important applications and engaged in trying to generalize the Krein-Milman's theorem. The few papers in this direction in the last two decades of his life culminate with the idea of the so-called *topological induction*, which leads to a substantial general-

ization of the Krein-Milman's theorem. This idea was reported at the seminar of Gustave Choquet (1915–2006) and was presented in a paper with proofs added by Mme Michèle Dehen. After achieving his goal, to find a more general instrument, Tagamlitzki, however, was not fully satisfied without finding interesting examples of effective applications of this instrument.

The volume ends with a paper on the results of Yaroslav Tagamlitzki on the *Diagonal Principle*, compiled from his manuscripts by Dimiter Skordev, and a paper on the *Topological Induction*, recorded by Ognyan Kounchev from Tagamlitzki's lectures during the academic year 1974/1975.

Blagovest Sendov